(Pages : 4) **E – 1705**

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme under CBCSS
MATHEMATICS
Core
MM 1643 : Complex Analysis – II
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

All Saints' College Trivandrum

SECTION – A

All the **first 10** questions are **compulsory**. **Each** carries **1** mark.

1. Define power series.
2. Find the circle of convergence of the power series $\sum_{n=0}^{\infty} z^n$.
3. Find the set of all singularities of $f(z) = |z|^2$.
4. Define essential singularity of a complex function.
5. Find the poles of $\frac{z + 2z^2}{\sin z}$.
6. Find the residue of $f(z) = \frac{z - \sin z}{z}$ at $z = 0$.
7. Find the order of zero of $f(z) = (ze^z - ze^z\cos z) - (z - z\cos z)$ at $z = 0$.
8. Find the Cauchy principal value of $\int_{-\infty}^{\infty} x\,dx$.
9. If f(z) is complex function having a pole of order 5 at $z = 0$. Then what type of singularity have the function $z^5 f(z)$?
10. What is the principal part of the Laurent's series expansion of $\frac{e^z}{z}$ at $z = 0$.

P.T.O.

SECTION – B

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. If f is analytic in D = D(;r) and if the closed rectangle R and the point a are both contained in D and C represents the boundary of R, then prove that $\int_C \frac{f(z)-f(a)}{z-a}dz = 0$.

12. State Cauchy's integral formula.

13. Evaluate $\int_C \frac{z^2+1}{(z^2-1)^2}dz$, where C is the circle $|z-1| = 1$ taken in counter clockwise direction.

14. Evaluate $\int_C z^2 \sin\left(\frac{1}{z}\right)dz$, where C is the circle $|z| = 1$ oriented in counter clockwise direction.

15. Find the residues of $\frac{z+1}{z^2(z-2)}$ at its poles.

16. If f is a complex function which is analytic in and on positively oriented circle C_R centered at z_0 with radius R and if M_R denotes the maximum of $|f(z)|$ on C_R then show that $|f^n(z_0)| \leq \frac{n!M_R}{R^n}$.

17. Find $\int_C \frac{1}{z^2+9}$, where C is the circle $|z-i| = 3$ oriented positively.

18. Find power series representation of sinhz in powers of Z.

19. Find the residue of f(z) = tanz at $z = \frac{\pi}{2}$.

20. If p and q are two complex functions, which are analytic at a point z_0 so that $p(z_0) \neq 0$ and q has a zero of order m at z_0. Show that $\frac{p(z)}{q(z)}$ has a pole of order m at z_0.

21. Evaluate $\int_0^\infty \frac{dx}{x^2+1}$.

22. Show that $\sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi}{6}$.

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SECTION – C

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each** :

23. Let C be a simple closed contour, described in the positive sense in the z-plane and if $g(z) = \int_C \frac{s^3 + 2s}{(s-z)^3} ds$, then find the value of $g(z_0)$ for z_0 interior to C and exterior to C.

24. Evaluate $\int_C \frac{dz}{(z^2+1)(z^2-4)}$, where C is the circle $|z| = \frac{3}{2}$ oriented in counter clockwise direction.

25. State and prove Cauchy's residue theorem.

26. Evaluate $\int_C \frac{e^z}{(z-1)^2(z^2+9)} dz$, where C is the circle $|z - 3i| = 1$ oriented in counter clockwise direction.

27. Evaluate the improper integral $\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)}$.

28. Evaluate $\int_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+1)^2}$.

29. Find the residue of $f(z) = \frac{1}{z(z-2)^4}$ at its poles and hence find $\int_C f(z)dz$ where C is the positively oriented circle $|z - 2| = 1$.

30. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \frac{-1}{2} + \frac{\pi}{2}\left(\frac{e^{2\pi}+1}{e^{2\pi}+1}\right)$.

31. Show that $\sum_{n=0}^{n} \binom{3n}{n} \frac{1}{8^n} = \frac{5+3\sqrt{5}}{5}$.

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SECTION – D

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Evaluate $\int_C \frac{dz}{4z^2+1}$ where C is the circle $|z| = 1$ oriented in counter clockwise direction.

b) Evaluate $\int_C \frac{2z^2 - z - 2}{(z-2)^4} dz$ where C is the circle $|z| = 3$ oriented in counter clockwise direction.

c) Show that $\int_C \frac{e^{az}}{z^{n+1}} dz = \frac{2a^n \pi i}{n!}$ where C is the circle $|z| = 1$ oriented in counter clockwise direction.

33. a) Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if f(z) can be written in the form $f(z) = \frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and nonzero at z_0.

b) Show that $\int_C e^{\left(z+\frac{1}{z}\right)} dz = 2\pi i \left(\sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \right)$ where C is the circle $|z| = 1$ oriented in counter clockwise direction.

34. a) Evaluate the integral $\int_0^{\infty} \frac{x^2}{x^6+1}$.

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$ where $-1 < a < 1$.

35. a) Prove that if f is complex function having only finite number of poles $\{z_k\}$, then $\sum_{n=1}^{\infty} f(n) = -\sum_k \text{Res}(f(z)\pi \cot \pi z, z_k)$.

b) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2 + a^2)(x^2 + b^2)}$ where $a > b > 0$.

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